A capacitance equivalent to C_0, when the outer structure has a near circularity, is given by (1a).

D. Superposition

It will now be assumed that, when the transformed structure does not deviate far from circularity, the two compensations can be applied mutually independent of each other. This is because the first compensation is positional, while the second is one of magnitude. Thus, superposing the two compensations in (5) and considering polar coordinates (η, Θ, z) with respect to the original structure in the Z plane, the following expression describing the TEM wave is obtained

$$\begin{Bmatrix} \boldsymbol{E}_{\text{rms}} \\ \boldsymbol{H}_{\text{rms}} \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{a}_{u'} \\ \dfrac{\boldsymbol{a}_{v'}}{\eta_0} \end{Bmatrix} \frac{4a_r V_{\text{rms}} e^{-j\beta z}}{(-G)\sqrt{D'}} \cdot \quad (10)$$

Here, if (6) to (8) are represented in the form,

$$u,\ v,\ D = F_1,\ F_2,\ F_3(r,\theta)$$

and

$$R = \frac{F_e(\theta) - r_1}{F_c(\theta) - r_1}$$

then,

$$u',\ v',\ D' = F_1,\ F_2,\ F_3 \cdot \left[\left(\frac{\eta^2 + R' r_1 - r_1}{R'}\right), 2\Theta\right]$$

and

$$R' = \frac{F_e(2\Theta) - r_1}{F_c(2\Theta) - r_1}.$$

III. Force on the Inner Conductor

From the properties of the transformation (3) and the bipolar coordinate,[5] the magnitude of a force acting on the inner conductor can be deduced.

$$F = \frac{dw}{de_1} = \frac{\pi \epsilon V^2}{a_r G^2} \quad (11)$$

where

F = force per unit length of the line,
w = energy stored in the electric field,
V = voltage difference between the inner and the outer conductors.

A one-to-one correspondence shows that forces $\vec{F}$ and $\overleftarrow{F}$ act on the inner conductor for the full structure along the minor axis of the ellipse in a direction tending to compress the former. The resultant displacement of the inner conductor from the geometric center of the ellipse, in conformity with quarter symmetry, is zero, by virtue of the equal and opposite nature of $\vec{F}$ and $\overleftarrow{F}$.

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Discontinuity Effects in Single Resonator Traveling Wave Filters*

In a previous correspondence the exact frequency response of the single resonator traveling wave directional filter was presented assuming that all transmission lines had characteristic impedances equal to the terminating impedance of the network.[1] The purpose of this correspondence is to extend the previous work to take into account the case where the transmission lines connecting the parallel coupled lines have an arbitrary characteristic impedance Z_I. The resulting structure is shown in Fig. 1.

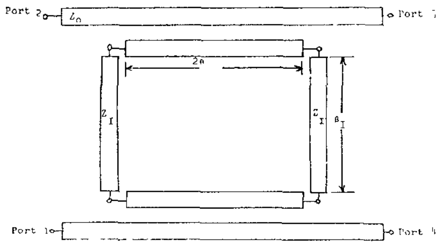

Fig. 1—Directional filter with arbitrary loop sides.

The symmetric-asymmetric excitation analysis is applied as discussed in the previous correspondence, with the transmission line matrix representing the loop sides plus corner equivalents replaced by

$$\begin{vmatrix} \cosh \alpha_I & Z_I \sinh \alpha_I \\ \dfrac{\sinh \alpha_I}{Z_I} & \cosh \alpha_I \end{vmatrix}.$$

In general $\alpha_I = j\beta_I$, and Z_I will be real so that the above becomes

$$\begin{vmatrix} \cos \beta_I & jZ_I \sin \beta_I \\ j\dfrac{\sin \beta_I}{Z_I} & \sin \beta_I \end{vmatrix}.$$

Making this change, carrying out the matrix multiplication, and calculating the transfer and reflection coefficients there is obtained,

$$T_S = 2\Bigg\{\Bigg[2\left(\cosh 2\alpha \cos\beta_I + \frac{\sin\beta_I \sinh 2\alpha}{2} \cdot \left(\frac{Z_0}{Z_I}\cot\theta - \frac{Z_I}{Z_0}\tan\theta\right)\right)\Bigg] + j\Bigg[\sin\beta_I\Bigg(\left(\frac{Z_0}{Z_I} + \frac{Z_I}{Z_0}\right)\cosh^2\alpha - \sinh^2\alpha\left(\frac{Z_I}{Z_0}\tan^2\theta + \frac{Z_0}{Z_I}\cot^2\theta\right)\Bigg) + \cos\beta_I \sinh 2\alpha(\tan\theta - \cot\theta)\Bigg]\Bigg\}^{-1} \quad (1)$$

$$T_A = 2\Bigg\{\Bigg[2\left(\cosh 2\alpha \cos\beta_I + \frac{\sin\beta_I \sinh 2\alpha}{2} \cdot \left(\frac{Z_I}{Z_0}\cot\theta - \frac{Z_0}{Z_I}\tan\theta\right)\right)\Bigg] + j\Bigg[\sin\beta_I\Bigg(\left(\frac{Z_0}{Z_I} + \frac{Z_I}{Z_0}\right)\cosh^2\alpha - \sinh^2\alpha\left(\frac{Z_I}{Z_0}\cot^2\theta + \frac{Z_0}{Z_I}\tan^2\theta\right)\Bigg) + \cos\beta_I \sinh 2\alpha(\tan\theta - \cot\theta)\Bigg]\Bigg\} \quad (2)$$

$$\Gamma_S = j\frac{T_s}{2}\Bigg\{\sin\beta_I\Bigg[\left(\frac{Z_I}{Z_0} - \frac{Z_0}{Z_I}\right)\cosh^2\alpha - \sinh^2\alpha\left(\frac{Z_0}{Z_I}\cot^2\theta - \frac{Z_I}{Z_0}\tan^2\theta\right)\Bigg] - \cos\beta_I \sinh 2\alpha(\cot\theta + \tan\theta)\Bigg\} \quad (3)$$

$$\Gamma_A = j\frac{T_A}{2}\Bigg\{\sin\beta_I\Bigg[\left(\frac{Z_I}{Z_0} - \frac{Z_0}{Z_I}\right)\cosh^2\alpha + \sinh^2\alpha\left(\frac{Z_I}{Z_0}\cot^2\theta - \frac{Z_0}{Z_I}\tan^2\theta\right)\Bigg] + \cos\beta_I \sinh 2\alpha(\cot\theta + \tan\theta)\Bigg\} \quad (4)$$

where

$$\cosh\alpha = \frac{Z_{0e} + Z_{00}}{Z_{0e} - Z_{00}}$$

and the other symbols are defined in Fig. 1.

Hence in the general case where $Z_I \neq Z_0$,

$$T_A \neq T_S$$
$$\Gamma_A \neq \Gamma_S.$$

Referring to the equations for the output voltages, the above results mean that:

1) A finite reflection will result at port 1.
2) Port 3 is not isolated.
3) A Butterworth response will not result at port 2.

In order to show the above more conclusively and to obtain an idea as to the effect of the ratio Z_I/Z_0, (1) through (4) were used to calculate the network response. Loaded Q values of 50 and 100 were used with Z_I/Z_0 as a parameter. It was also assumed that $2\theta = \beta_I$. The results of the calculations are plotted in Figs. 2–5 for $Q_L = 100$.

It is important to note that the computed results predict the experimentally observed fact that a misaligned filter of this type yields a double resonance response shape.

To obtain a knowledge of the effect of β_I, the responses were calculated with $2\theta/\beta_I = k$ as a parameter. It was assumed that β_I would have a linear frequency dependence over the limited range of electrical angle of the calculations. The results were plotted and it was found that the primary effect of having $k \neq 1$ is to shift the resonant frequency to

$$f = \frac{k}{k+1} f_0$$

where it has been assumed that

$$2\theta = \frac{\pi}{2} \frac{f}{f_0}.$$

* Based on part of the research work undertaken by Robert D. Standley in partial fulfillment of the requirements for the Ph.D. degree at Illinois Institute of Technology; Chicago, Ill.

[1] R. D. Standley, "Frequency response of str line traveling wave directional filters," IEEE Trans. on Microwave Theory and Techniques, vol. 11, pp. 264–265; July, 1963.

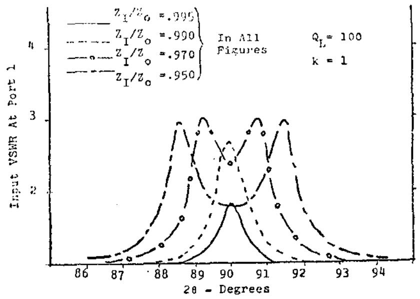

Fig. 2—Input VSWR as a function of Z_I/Z_0 ($Q_L=100$).

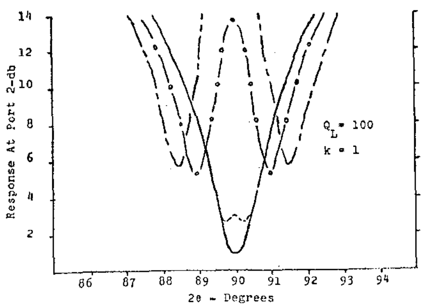

Fig. 3—Port 2 response ($Q_L=100$).

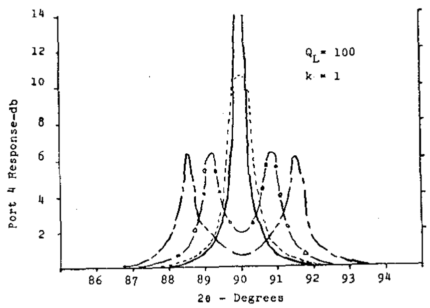

Fig. 4—Port 4 response ($Q_L=100$).

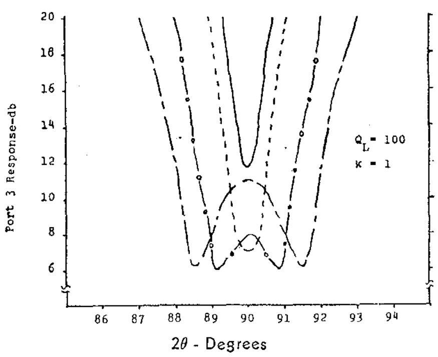

Fig. 5—Port 3 response ($Q_L=100$).

Plots such as those shown in Figs. 2–5 are of value in two possible ways. If a particular type of discontinuity such as a dielectric post exists in the loop sides, then a knowledge of the discontinuity equivalent circuit permits calculation of Z_I and β_I used above. Thus the discontinuity effects are readily taken into account, and the resulting frequency response is predictable. Secondly, plots of the above form can serve as aids to empirical adjustment by comparing the measured response to the predicted. In this case the required assumption is that the variation of Z_I is negligible over the frequency band of interest.

In summary, equations for the transmission and reflection coefficients for the single resonator traveling wave filter have been presented for the case where the loop sides have an arbitrary characteristic impedance. It has been shown that when $Z_I \neq Z_0$ ideal directional filter characteristics are not attainable.[2] The effects on frequency response are in qualitative agreement with those observed experimentally. Also $\beta_I \neq 2\theta$ cause a predictable frequency shift.

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[2] F. S. Coale, "A Traveling-Wave Directional," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-4, pp. 256–260; October, 1956.

A Wall-Current Detector for Use with Beam Waveguides*

A crystal detector mount which is well suited for use with beam waveguides[1] of either the refracting or reflecting type is based on the same concept as the wall-current detector of deRonde.[2] The geometry of one form of the detector is shown in Fig. 1.

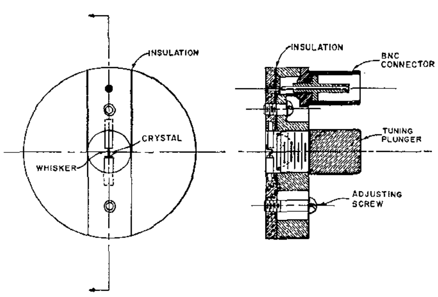

Fig. 1—Schematic drawing of a wall-current detector for use with beam waveguides.

Radiation incident on the detector with its magnetic intensity horizontally polarized causes a vertical surface current to flow, some of which passes through the rectifying junction mounted across the circular aperture. A tuning plunger mounted behind the rectifying junction is used in the conventional manner to tune the detector. A dielectric lens may be used directly in front of the detector to concentrate the incident radiation on the diode.

A detector of this type has been used for making measurements on a 70-Gc beam waveguide. Using relatively crude assembly techniques it is possible to construct units with an output voltage of the order of 15 millivolts per milliwatt of power in the radiation beam, which has a minimum cross-sectional area of approximately 6 square centimeters.

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[1] G. Goubau and F. Schwering, "On the Guided propagation of electromagnetic wave beams," IRE TRANS. ON ANTENNAS AND PROPAGATION, vol. AP-9, pp. 248–256; May, 1961.

[2] F. C. deRonde, "A Universal Wall-Current Detector," presented at Millimeter and Submillimeter Conf., January 7–10, 1963, Orlando, Fla.

Solution of the Fourier Heat Flow Equation in Waveguides Filled with Lossy Dielectrics*

The temperature distribution within a waveguide filled with a lossy dielectric, and propagating a specified mode, may be conveniently obtained by the Green's function method. Such a solution is useful for describing the temperature distribution in vacuum windows, for example.

The temperature distribution in the steady state is given by Fourier's equation

$$\kappa \nabla^2 T + p = 0 \tag{1}$$

where κ is the thermal conductivity and T the temperature of the material. A spontaneous source of heat power per unit volume p is supposed. In the example considered here, the heat power is presumed to be generated by some mechanism as molecular friction due to oscillation of the dipolar material attempting to align itself with the electric field of an RF wave. This heat will establish a thermal gradient in the dielectric material as it flows toward a heat sink, which will constitute a boundary condition on the differential equation. We will not consider temperature variations along the axis of the waveguide, which is equivalent to assuming that the loss per unit length along the axis is small. Therefore we will only require a solution of Poisson's equation (and a Green's function) in two dimensions.

To illustrate the explanation we consider the case of the cylindrical TE-11 mode, for which the field description is given to sufficient accuracy by the solution of the homogeneous wave equation (1):

$$E_r = \frac{2E_0}{k_c r} J_1(k_c r) \sin \phi j \exp j(\omega t - \beta z)$$

$$E_\phi = 2E_0 J_1'(k_c r) \cos \phi j \exp j(\omega t - \beta z) \tag{2}$$

where E_0 is the maximum field intensity at the origin. Thus, the heat power source function is given by

$$p(r, \phi) = \frac{\sigma E^2}{2} = \frac{\sigma E_0^2}{2} \left[J_0^2(k_c r) + J_2^2(k_c r) - 2J_0(k_c r) J_2(k_c r) \cos 2\phi \right] \tag{3}$$

* Received June 17, 1963; revised manuscript received August 7, 1963.